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Role of Modified Chaplygin Gas as an Unified Dark Matter-Dark Energy Model in Collapsing Spherically Symmetric Dust Cloud

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Abstract In this work gravitational collapse of a spherical dust cloud in the background of unified dark matter-dark energy model in the form of modified Chaplygin gas is studied. It is found that invisible matter (dark matter-dark energy) alone in the form of modified Chaplygin gas forms black hole. Also when both components of the fluid are present then the collapse favours the formation of black hole in cases the invisible matter dominates over ordinary dust. The conclusion is totally opposite to the usually known results.

Keywords Collapse · Dark matter · Chaplygin gas

1 Introduction

Recent observations of the anisotropy of the Cosmic Microwave Background Radiation [1–5] and of the type Ia Supernovae SN 1997H redshift [6–11] indicate that the universe is flat with a present positive acceleration preceeded by a period of deceleration. To match with these observational evidences ordinary (baryonic) matter or radiation is insufficient, a significant part of the energy density of the universe should be an extraordinary non-baryonic matter (dark matter) and energy (dark energy). Although there are broad interest in dark matter and dark energy still we know very little about their physical properties. Also it is not clear whether they are two separate substances.

However, from a multitude of observations, there is a strong evidence that dark matter (DM) is of the order of 25% of the critical density while dark energy (DE) constitutes about 2/3 of the critical density [11–15]. The uniformly distributed dark energy with a negative pressure has came to dominate the universe recently (at redshifts $z \lesssim 1$) and is responsible

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for the present accelerating phase of the universe. Several mechanisms have been proposed during the last few years to explain the above recent observational evidences and among them, the single-component perfect fluid having exotic equation of state known as Chaplygin gas [16–29] is of great interest. A more general form, known as modified Chaplygin gas obeys the following caloric equation of state [16–20]

$$p = \gamma\rho - \frac{B}{\rho^{\alpha}}, \quad B > 0,\ 0 < \alpha < 1. \tag{1}$$

The three parameters γ, β and α are universal positive constants. This equation of state with $\gamma = 0$ and $\alpha = 1$ (known as Chaplygin gas) has been first considered to describe lifting forces on a plane wing in aerodynamics process [21]. Its generalization with $\gamma = 0$, $\alpha > 0$ is known as generalized Chaplygin gas (GCG), first introduced by Kamenshchik et al. [22] and Bento et al. [23, 24]. In GCG model, at low energy density, the fluid pressure is negative and constant while at high energy density the model behaves as an almost pressurelessfluid. However, to interpolate states between standard fluids at high pressures at high energy densities and constant negative pressure at low energy densities, one must need the modified Chaplygin gas model, an extension of GCG model [16, 25]. In particular, for $\gamma = 1/3$, the modified Chaplygin gas model describes the evolution from that of radiation era at early time to Λ CDM era at late times where the fluid (behaves as a cosmological constant) has almost constant energy density and as a result there will be an accelerated expansion [16, 17, 25]. Another point that goes against the GCG model is the velocity of sound. In GCG model, the velocity of sound is negligible at early times and it approaches the speed of light at late time limit. The sound speed of the order of speed of light is not compatible [26] in cosmic evolution.

Moreover the Modified Chaplygin gas model is interesting from phenomenological point of view and it can be motivated by brane world interpretation [23, 24]. Also this model is consistent with various classes of cosmological tests namely gravitational lensing [27, 28], gamma-ray bursts [29] and above mentioned observations. Further, in comparison to other competing candidates to explain the overwhelming energy density of the present universe, the present modified Chaplygin gas model (also generalized Chaplygin gas model) is naturally constrained through Cosmological observables [26].

Further, for low density, the present model is very similar to generalized Chaplygin gas model so the equation of state is that of a polytropic gas [25] with negative index. Thus it is possible to have astrophysical implications of the present model with an alternative way of restricting the parameters [26].

In this paper, gravitational collapse of a spherically symmetric dust cloud in the background of modified Chaplygin gas model (a unified dark matter-dark energy model) is considered. The issue of collapse in the context of the unified model has been recently the subject of various studies [27, 28]. Now the energy-momentum tensor of the combined cloud is

$$T_i^j = (\rho_V + \rho + p)u_i u^j - p\delta_i^j \tag{2}$$

where ρ_V is the energy density of the ordinary dust cloud and (ρ, p) are related by (1).

We consider, gravitational collapse of a spherically symmetric dust star in the background of modified Chaplygin gas model. Let us assume, first divide the space-time into three different regions, Σ and $V^{\pm}$, where Σ is the boundary of the star, V^- and V^+ are interior and exterior of the star respectively.

The Einstein equations for spherical space-time inside the star with line-element

$$ds_-^2 = dt^2 - a^2(t)(dr^2 + r^2 d\Omega^2) \tag{3}$$

are given by

$$3\frac{\dot{a}^2}{a^2} = \kappa(\rho_V + \rho) \tag{4}$$

and

$$2\frac{\ddot{a}}{a} + \frac{\dot{a}^2}{a^2} = \kappa p. \tag{5}$$

Now, if $Q(t)$ denotes the interaction between dark matter and dark energy then from the conservation law $T^{\nu}_{\mu;\nu} = 0$ one gets

$$\dot{\rho}_V + 3\frac{\dot{a}}{a}\rho_V = Q \tag{6}$$

and

$$\dot{\rho} + 3\frac{\dot{a}}{a}(\rho + p) = -Q. \tag{7}$$

If $\Sigma : r = r_\Sigma$ denotes the boundary of the spherical collapsing cloud then on Σ

$$ds^2_\Sigma = d\tau^2 - R^2(\tau)d\Omega^2 \tag{8}$$

where $\tau = t$ and $R(\tau) = r_\Sigma a(\tau)$ is called the area radius. The metric outside the collapsing star in general can be written in the form [30, 31]

$$ds^2_+ = A^2(T, R)dT^2 - B^2(T, R)(dR^2 + R^2 d\Omega^2).$$

The surface Σ in the exterior coordinates can be expressed as $R = R_0(T)$. Israel's junction conditions on the boundary have been discussed in details by Cai and Wang [30, 31]. Once dependence of A and B on T and R, it is possible to determine the time evolution of T, R_0, A and B along the hypersurface Σ uniquely.

For gravitational collapse $\dot{a} < 0$ and $R(t, r) \equiv ra(t)$ denotes the geometric radius of two-spheres, $(t, r) =$ constant. The mass function inside the radius r at the moment t is defined by

$$m(t, r) = \frac{1}{2}R(1 + R_{,\alpha}R_{,\beta}g^{\alpha\beta}) = \frac{1}{2}r^3 a\dot{a}^2.$$

Thus the total mass of the collapsing cloud is

$$M(\tau) = m(\tau, r_\Sigma) = \frac{1}{2}R(\tau)\dot{R}^2(\tau). \tag{9}$$

If τ_{AH} be the time instant at which the whole cloud starts to be trapped then

$$R_{,\alpha}R_{,\beta g^{\alpha\beta}}|_{\tau=\tau_{AH}} = 0, \quad \text{i.e., } \dot{R}^2(\tau_{AH}) = 1. \tag{10}$$

As it is natural to assume the cloud to be untrapped initially $(t = t_i)$ so one should have

$$\dot{R}^2(\tau = \tau_i) < 1. \tag{11}$$

Note that if the condition (11) holds throughout the collapsing process then the collapse will not produce black holes. In the following two sections, collapsing process will be studied when there is only Chaplygin gas as the collapsing fluid and then a combination of ordinary matter and Chaplygin gas both with and without interaction. The paper ends with some conclusive remarks.

2 Gravitational Collapse of Dark Matter-Dark Energy as Chaplygin Gas Model

This section deals with gravitational collapse of dark matter-dark energy in the form of Chaplygin gas. From the conservation equation (7), integrating once one gets

$$\rho = \left[\frac{B}{1+\gamma} + \frac{C}{a^{3(1+\gamma)(1+\alpha)}}\right]^{\frac{1}{1+\alpha}}, \quad (\gamma \neq -1) \tag{12}$$

with C is the constant of integration.

Now substituting this expression for ρ into the Friedman equation (4) and integrating the scale factor can be obtained as

$$a^{\frac{3(1+\gamma)}{2}} {}_2F_1\left[x, x, 1+x, -\frac{B}{C(1+\gamma)} a^{\frac{3(1+\gamma)}{2x}}\right] = \frac{\sqrt{3\kappa}}{2}(1+\gamma)C^x(t_0 - t) \tag{13}$$

where $x = \frac{1}{2(1+\alpha)}$ and ${}_2F_1$ is the hypergeometric function.

The expressions for the related physical parameters are

$$\dot{R}(\tau) = -R_0 a^{-\frac{3(1+\gamma)}{2}}\left[C + \frac{B}{1+\gamma} a^{3(1+\alpha)(1+\gamma)}\right]^{\frac{1}{2(1+\alpha)}}, \tag{14}$$

$$M(\tau) = \frac{1}{2} R_0^2 r_\Sigma a^{-3\gamma}\left[C + \frac{B}{1+\gamma} a^{3(1+\alpha)(1+\gamma)}\right]^{\frac{1}{(1+\alpha)}}. \tag{15}$$

One may note that as $t \to t_0$

$$a^{\frac{3(1+\gamma)}{2}} \simeq \frac{\sqrt{3\kappa}}{2}(1+\gamma)C^{\frac{1}{2(1+\alpha)}}(t_0 - t) \sim 0.$$

Also using the relation [16]

$$\begin{aligned} {}_2F_1[a,b,c;z] = {} & \frac{\Gamma(c)\Gamma(b-a)}{\Gamma(b)\Gamma(c-a)}(-z)^{-a} {}_2F_1\left[a, 1-c+a, 1-b+a; \frac{1}{z}\right] \\ & + \frac{\Gamma(c)\Gamma(a-b)}{\Gamma(a)\Gamma(c-b)}(-z)^{-b} {}_2F_1\left[b, 1-c+b, 1-a+b; \frac{1}{z}\right] \end{aligned} \tag{16}$$

one gets the limiting value of

$$a^{\frac{3(1+\gamma)}{2}} {}_2F_1\left[\frac{1}{2(1+\alpha)}, \frac{1}{2(1+\alpha)}, 1 + \frac{1}{2(1+\alpha)}, -\frac{B}{C(1+\gamma)} a^{3(1+\alpha)(1+\gamma)}\right]$$

as

$$\frac{1}{1+\alpha}\left[\frac{C(1+\gamma)}{B}\right]^{\frac{1}{2(1+\alpha)}}$$

when a is very large. Thus if $t \to t_s$ as $a \to \infty$ then from (13)

$$t_s = t_0 - \frac{2}{\sqrt{3\kappa}(1+\alpha)(1+\gamma)}\left(\frac{1+\gamma}{B}\right)^{\frac{1}{2(1+\alpha)}}. \tag{17}$$

The limiting value of the physical parameters are

$$\tau \to \tau_s : \rho \to \left[\frac{B}{1+\gamma}\right]^{\frac{1}{1+\alpha}}, \quad \dot{R} \to \begin{cases} -\infty, & \text{for } \gamma > -5/3, \\ 0, & \text{for } \gamma \le -5/3, \end{cases} \quad M(\tau) \to \infty,$$

$$\tau \to \tau_0 : \rho \to \infty, \quad \dot{R} \to -\infty, \quad M(\tau) \to \infty.$$

Thus if the collapse starts at an instant close to τ_s then for $\gamma > -5/3$, initially the collapsing system is trapped and in course of the collapsing process it gets untrapped (provided the maximum value of $\dot{R}$ is greater than -1) and then again it is trapped and black hole forms. However, for $\gamma \le -5/3$, the system is initially untrapped and as it approaches to the singularity at $\tau = \tau_0$, it gets trapped and leads to the formation of a black hole. Thus dark matter-dark energy alone in the form of Chaplygin gas favours formation of black hole.

3 Collapsing Process under the Joint Influence of Ordinary Matter and the Combination of Dark Matter and Dark Energy

This section is divided into two parts. In the first case, the interaction $Q(t)$ is neglected while in the second case, the influence of $Q(t)$ is considered.

Case 1 *Interaction is Neglected* : $Q(t) = 0$

Here the conservation equation for ρ_V gives

$$\rho_V = \frac{\rho_0}{a^3}, \quad \rho_0 > 0, a \text{ constant.} \tag{18}$$

The expressions for $\dot{R}(\tau)$ and $M(\tau)$ are

$$\dot{R}(\tau) = -R_0 a^{\frac{1}{2}} \left[\rho_0 + a^{-3\gamma}\left\{C + \frac{B}{1+\gamma} a^{3(1+\alpha)(1+\gamma)}\right\}^{\frac{1}{1+\alpha}}\right]^{\frac{1}{2}} \tag{19}$$

and

$$M(\tau) = \frac{1}{2} R_0^2 r_\Sigma a^2 \left[\rho_0 + a^{-3\gamma}\left\{C + \frac{B}{1+\gamma} a^{3(1+\alpha)(1+\gamma)}\right\}^{\frac{1}{1+\alpha}}\right] \tag{20}$$

with $R_0 = r_\Sigma \sqrt{\frac{\kappa}{3}}$.

As the integral in (19) can not be evaluated in general, so only the approximate forms for 'a' may be obtained for small and large 'a'. However, one can determine the behaviour of the physical parameters in these two limits (namely, $a \to 0$ and $a \to \infty$) as follows:

$$a \to 0 : \rho_V \to \infty, \quad \rho \to \begin{cases} \infty, & \text{if } 1+\gamma > 0, \\ a \text{ constant} & \text{if } 1+\gamma < 0, \end{cases} \quad \dot{R} \to \begin{cases} 0, & \text{if } \gamma < \frac{1}{3}, \\ -\infty, & \text{if } \gamma > \frac{1}{3}, \\ -\mu, & \text{if } \gamma = \frac{1}{3}, \end{cases}$$

$$M \to \begin{cases} 0, & \text{if } \gamma < \frac{2}{3}, \\ \infty, & \text{if } \gamma > \frac{2}{3}, \\ a \text{ constant}, & \text{if } \gamma = \frac{2}{3}, \end{cases} \quad \mu = R_0 C^{\frac{1}{2(1+\alpha)}}. \tag{21}$$

$$a \to \infty : \rho_V \to 0, \quad \rho \to \begin{cases} a \text{ constant}, & \text{if } 1+\gamma > 0, \\ \infty, & \text{if } 1+\gamma < 0, \end{cases} \quad \dot{R} \to -\infty, \quad M \to \infty. \tag{22}$$

Thus $a = 0$ is always a singularity of the space-time and it is covered by an apparent horizon for $\gamma \geq 1/3$ (provided $\mu > 1$) while the singularity is naked for $\gamma < 1/3$. On the otherhand, $a = \infty$ may be singular if $\gamma < -1$ and always black hole will form.

Case 2 *Gravitational Collapse with Interaction*

For interaction using the idea of Cai and Wang [30, 31], we have assumed the ratio of dark matter-dark energy density and ordinary matter density as

$$\frac{\rho}{\rho_V} = Aa^{3n} \tag{23}$$

with $A > 0$ and n as arbitrary constants. then solving the conservation equations (6) and (7) one obtains

$$\begin{aligned} \rho_t^{\alpha+1} = {} & \frac{(\alpha+1)B}{[\alpha(n-1)-1]} \frac{(Aa^{3n})^{\frac{2}{n}(\alpha+1-n\alpha)-(\alpha+1)}}{(Aa^{3n}+1)^{\frac{2}{n}(\alpha+1-n\alpha)+\gamma(\alpha+1)}} \\ & \times {}_2F_1\left[\frac{1+\alpha-n\alpha}{n}, \frac{1+n+\alpha+\gamma+\alpha\gamma}{n}, \frac{1+n+\alpha-n\alpha}{n}, \frac{Aa^{3n}}{1+Aa^{3n}}\right] \\ & + z_0[Aa^{3n}(1+Aa^{3n})^{\gamma}]^{-(\alpha+1)} \end{aligned} \tag{24}$$

where $\rho_t = \rho + \rho_V$ and using (23) one gets

$$\rho = \frac{Aa^{3n}\rho_t}{1+Aa^{3n}}, \qquad \rho_V = \frac{\rho_t}{1+Aa^{3n}}. \tag{25}$$

Hence from the conservation equation (6) and the Friedman equation (4) the expression for the interaction is

$$Q(t) = -\frac{3(\gamma+n)Aa^{3n}\rho_t}{(1+Aa^{3n})^2}\frac{\dot{a}}{a} + \frac{3B(1+Aa^{3n})^{\alpha-1}}{\rho_t^{\alpha}A^{\alpha}a^{3n\alpha}}\frac{\dot{a}}{a} \tag{26}$$

where

$$\begin{aligned} \frac{\dot{a}}{a} = {} & -\frac{\rho_0}{a^{\frac{3n}{2}}(1+Aa^{3n})^{\frac{\gamma}{2}}}\Bigg[\frac{a^{6(\alpha+1-n\alpha)}A^{\frac{2}{n}(\alpha+1-n\alpha)}}{(1+Aa^{3n})^{\frac{2}{n}(\alpha+1-n\alpha)}} \\ & \times {}_2F_1\left[\frac{1+\alpha-n\alpha}{n}, \frac{1+n+\alpha+\gamma+\alpha\gamma}{n}, \frac{1+n+\alpha-n\alpha}{n}, \frac{Aa^{3n}}{1+Aa^{3n}}\right] + z_1\Bigg]^{\frac{1}{2(\alpha+1)}} \end{aligned} \tag{27}$$

with

$$\rho_0 = \sqrt{\frac{\kappa}{3A}}\left[\frac{(\alpha+1)B}{\alpha(n-1)-1}\right]^{\frac{1}{2(\alpha+1)}}, \qquad z_1 = z_0\left[\frac{(\alpha+1)B}{\alpha(n-1)-1}\right]^{-\frac{1}{2(\alpha+1)}}.$$

The equation (27) can be written in the integral form as

$$\int \frac{(1+Ay^2)^{\frac{\gamma}{2}}dy}{[\frac{(Ay^2)^{\frac{2}{n}(\alpha+1-n\alpha)}}{(1+Ay^2)^{\frac{2}{n}(\alpha+1-n\alpha)}}{}_2F_1[\frac{1+\alpha-n\alpha}{n},\frac{1+n+\alpha+\gamma+\alpha\gamma}{n},\frac{1+n+\alpha-n\alpha}{n},\frac{Ay^2}{1+Ay^2}]+z_1]^{\frac{1}{2(\alpha+1)}}}$$
$$= -y_0(t-t_0) \tag{28}$$

with $y = a^{3n/2}$ and $y_0 = \frac{3n}{2}\rho_0$. The expression for mass function and $\dot{R}$ are

$$M(\tau) = \frac{r_\Sigma^3\rho_0^2}{2a^{3n-1}(1+Aa^{3n})^\gamma}\left[\frac{a^{6(\alpha+1-n\alpha)}A^{\frac{2}{n}(\alpha+1-n\alpha)}}{(1+Aa^{3n})^{\frac{2}{n}(\alpha+1-n\alpha)}}\right.$$
$$\left.\times {}_2F_1\left[\frac{1+\alpha-n\alpha}{n},\frac{1+n+\alpha+\gamma+\alpha\gamma}{n},\frac{1+n+\alpha-n\alpha}{n},\frac{Aa^{3n}}{1+Aa^{3n}}\right]+z_1\right]^{\frac{1}{\alpha+1}} \tag{29}$$

and

$$\dot{R}(\tau) = -\frac{\rho_0 r_\Sigma}{a^{(\frac{3n}{2}-1)}(1+Aa^{3n})^{\frac{\gamma}{2}}}\left[\frac{a^{6(\alpha+1-n\alpha)}A^{\frac{2}{n}(\alpha+1-n\alpha)}}{(1+Aa^{3n})^{\frac{2}{n}(\alpha+1-n\alpha)}}\right.$$
$$\left.\times {}_2F_1\left[\frac{1+\alpha-n\alpha}{n},\frac{1+n+\alpha+\gamma+\alpha\gamma}{n},\frac{1+n+\alpha-n\alpha}{n},\frac{Aa^{3n}}{1+Aa^{3n}}\right]+z_1\right]^{\frac{1}{2(\alpha+1)}}. \tag{30}$$

The above expressions for the physical parameters show the following limiting behaviour

$$\rho_t \sim \begin{cases} a^{-3n}, & a\to 0,\\ a^{-3n(\alpha+1)(\gamma+1)}, & a\to\infty, \end{cases} \qquad \rho \sim \begin{cases} a \text{ constant}, & a\to 0,\\ a^{-3n(\alpha+1)(\gamma+1)}, & a\to\infty, \end{cases}$$

$$\rho_V \sim \begin{cases} a^{-3n}, & a\to 0,\\ a^{-3n(\alpha+1)(\gamma+1)}, & a\to\infty, \end{cases} \qquad \dot{R}(\tau) \sim \begin{cases} -a^{1-\frac{3n}{2}}, & a\to 0,\\ -a^{1-\frac{3n}{2}(1+\gamma)}, & a\to\infty, \end{cases}$$

$$M(\tau) \sim \begin{cases} a^{1-3n}, & a\to 0,\\ a^{1-3n(\gamma+1)}, & a\to\infty. \end{cases}$$

It is to ne noted that $a = 0$ is always a singularity of the space-time but $a = \infty$ is singularity if $1+\gamma < 0$.

The above integral in (28) is solvable for the (choice $\alpha = 1$, $n = 2$ and one gets) restriction $1+\alpha = n\alpha$ and one gets

$$\frac{y}{(1+z_1)^{\frac{1}{4}}}{}_2F_1\left[\frac{1}{2},-\frac{\gamma}{2},\frac{3}{2},-Ay^2\right] = -y_0(t-t_0). \tag{31}$$

Thus in the limit as $a\to 0$, one finds

$$a \sim a_0(t_0-t)^{\frac{1}{3}}, \quad a_0 = y_0^{\frac{2}{3}}(1+z_1)^{\frac{1}{6}} \quad \text{i.e., } a\sim 0 \text{ as } t\to t_0.$$

Further, using the property of the hypergeometric function (see (16)), for large a, the solution (31) approximates to

$$a^{1+\gamma} \sim a_1(t_0 - t), \quad a_1 = \frac{4a_0}{(1+\gamma)A^{\frac{\gamma}{2}}}, \text{ for } \gamma > -1$$

and

$$\frac{\sqrt{\pi}\Gamma(-\frac{\gamma+1}{2})}{2\sqrt{A}(1+z_1)^{\frac{1}{4}}\Gamma(-\frac{\gamma}{2})} = -y_0(t_s - t_0) \quad \text{i.e.,}$$

$$t_s = t_0 - \frac{\sqrt{\pi}\Gamma(-\frac{\gamma+1}{2})}{2\sqrt{A}a_0^3\Gamma(-\frac{\gamma}{2})}, \text{ for } \gamma \leq -1.$$

The limiting value of the physical parameters show that if $n < 2/3$ then the space-time collapses to a naked singularity while black hole will form for $n > 2/3$. However, the singularity at $a = \infty$ always corresponds to a black hole solution for $1 + \gamma < 0$.

4 Conclusion

The paper deals with gravitational collapse of a spherically symmetric homogeneous and isotropic fluid having finite radius. The fluid has two component—one component is in the form of ordinary dust and the other, a combination of dark matter and dark energy component is the modified Chaplygin gas model.

When the collapsing fluid is only in the form of modified Chaplygin gas (the dark energy) then the collapse always leads to the formation of a black hole. But there is some peculiarity for $\gamma > -5/3$. Initially, the space-time is trapped and during the evolution it gets untrapped and again it is covered by an apparent horizon. This feature is interpreted by Cai and Wang [30, 31] as the evaporation of a white hole by ejecting matter which again re-collapse to form a black hole. Note that the collapsing dark energy in the form of Chaplygin gas can alone form black holes unlike the dark energy model of Cai and Wang [30, 31] is not in favour of black hole.

Section 3 deals with collapsing fluid having both components with or without interaction. In both cases it is also found that when the unified dark matter density dominates over the ordinary matter energy density then the collapse favours formation of black hole. Further, the expression for the interaction parameter has two terms of which the first one is identical to that of Cai and Wang [30, 31] while the second term, due to the Chaplygin gas having negative sign reduces the interaction parameter. Therefore, from the above study, one may conclude that the unified dark matter is not always against the formation of black holes, it favours the formation of apparent horizon in some cases.

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